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"Nonlinear Wave Propagation"

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University of Colorado - Boulder  
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**Abstract**

This research program involves the basic understanding and applications of nonlinear wave propagation. A number of significant results have been obtained. Contributions include the following. The analysis and computation of solitons and their interaction properties in nonlinear optical fibers; solutions and general behavior of a class of physically significant multidimensional and novel nonlinear wave equations; studies of discrete nonlinear evolution systems and numerically induced chaos in a class of integrable nonlinear systems. This report also includes a list of recent publications and preprints.

## **Summary of Research Activities**

During the past two years there have been eight journal publications, three more have been accepted for publication in journals, seven book chapters have been published and a monograph "Solitons, Nonlinear Evolution Equations and Inverse Scattering" was published by Cambridge University Press. In addition there are nine Program in Applied Mathematics Reports (preprints of the Program) which have been submitted for publication and the PI has been invited to give fourteen lectures at universities and international conferences throughout the world. The past two years has been a very active and productive period in this research program. A brief overview and description of our research investigations are given below. Full details are included in the publications.

## **Soliton Propagation in Nonlinear Optics**

In the application of optical data communication, linear transmission systems work well over short distances or with relatively low data rates. However, chromatic dispersion, dissipation and nonlinear effects seriously limit the distances over which the linear transmission is viable. In practice, numerous electronic repeaters to regenerate linear signals are required in order to make long distance data communication effective. Using nearly linear waves has a number of disadvantages and researchers have studied the possibility of developing practical transmission systems which use the underlying nonlinearity of the optical fiber as a critical feature. The net result is that solitons have proven to be an important technological mechanism for data communication. Much of the research has centered on sending widely separated solitons with many equally spaced amplifiers to counteract dissipation. The amplifiers are considerably easier to produce than electronic repeaters and being optical in nature they have the potential of allowing nonlinear systems to transmit information at much larger data rates than those of linear systems.

In many of the important applications, the dominant underlying nonlinear equation is one of the integrable nonlinear systems which the PI and colleagues have intensively studied with the support of the Air Force. In the mathematical formulation, solitons are associated with eigenvalues of certain linear systems and without damping or amplification, the eigenvalues are preserved in time. In fact even with perturbations, it is known that these eigenvalues only evolve slowly in time. One of the realistic models governing singly polarized optical fibers is the nonlinear Schrödinger equation,(NLS). With the NLS equation, the real part of the eigenvalue is related to the carrier frequency of the optical wave and the imaginary part of the eigenvalue is associated with the amplitude. In our research we are interested in wavelength division multiplexing (WDM), which is the study of sending many solitons in a given window of time. The goal is to transmit a group of solitons which can be measured easily at the receiving station. In other words, we are really interested in transmitting and receiving a collection of eigenvalues over long distances.

In this project we have collaborated with a group led by Professor Jon Sauer of the Optoelectronics Computing Center at the University of Colorado. Mathematically and physically the problems that must be addressed are considerably more difficult when multisolitons are present. One difficulty is the need to distinguish separate solitons (eigenvalues) when they overlap in physical space. In our research, we have shown both analytically and numerically that for two solitons the power spectra (amplitudes of the Fourier transforms) always remain well separated so long as the carrier frequencies (the imaginary parts of the eigenvalues) of the solitons are well separated. These calculations pave the way to understand N soliton WDM and provides a basis for how a suitable perturbation theory with damping and frequency effects could be developed.

### **Multidimensional and Novel Nonlinear Wave Equations**

The well known integrable nonlinear wave equations of physical significance include: the Korteweg-deVries (KdV) equation and its 2+1 dimensional analogue: the Kadomtsev-Petviashvili equation; the nonlinear Schrödinger equation and its 2+1 analogue: the Davey-Stewartson equation; and the 1+1 and 2+1 dimensional N wave interaction equations. These equations have been extensively studied by us in a variety of contexts. We are now studying important discrete nonlinear wave systems such as the 2+1 Toda equation which reduces to the well known Toda lattice equation in 1+1 dimension. The 2+1 Toda equation admits a variety of interesting phenomena depending on the choice of signs in the governing equations. In the choice of signs where the initial value problem is well posed, there are two cases, one of which admits linearly unstable waves. The inverse scattering analysis in the unstable case results in a coupled nonlocal Riemann-Hilbert and DBAR system. For the other choice of signs, the linearized Toda equation is ill-posed. Nevertheless, interesting boundary value problems can be formulated and solved. In one of the subcases the boundary value problem requires that a radiation condition be imposed in order to obtain a unique solution. This radiation condition is an analogue of the well known Sommerfeld radiation condition required in linear problems, such as the Helmholtz equation.

We have been led to study a nonlinear differential-delay Toda equation, which is a reduction of the 2+1 Toda equation. Differential-delay problems arise frequently in physical problems. This is the first time we have seen such an equation in the field of soliton theory. In the study of the Toda differential-delay equation, the analysis of the associated inverse scattering problem is quite complicated. The inverse scattering problem is novel. It requires one to solve an infinitely coupled system of Riemann-Hilbert problems on adjacent strips. In previous applications the inverse problem involved solving a Riemann-Hilbert problem in half-planes or in sectors. The soliton solutions also arise in a novel fashion and they are described as bound states on a suitably branched Riemann surface.

Wave collapse has been an area of active study by us as well. It is well known that the multidimensional (e.g. 2+1) cubic nonlinear Schrödinger equations (arising in many areas of physics; e.g. nonlinear optics) has solutions which blow up in finite time. It is less well known that one dimensional nonlinear (quintic and higher nonlinearity) analogues of the Korteweg-deVries equation also admit solutions which tend to infinity in finite time. We have recently shown that nonlinear versions (cubic and higher nonlinearity) of the Kadomtsev-Petviashvili (KP) equations possess similar blow-up phenomena. We have studied this problem both analytically and numerically. Indeed, we have found that in the numerical calculations one must impose appropriate constraints which are inherent in KP systems and which we had studied earlier in the context of the integrable KP equation (the quadratically nonlinear system).

### **Numerical Chaos--Truncation and Roundoff**

In our research we have been studying moderate to long time numerical experiments involving integrable systems, most notably the nonlinear Schrödinger, sine-Gordon and modified Korteweg-de Vries equations with periodic boundary conditions. In our earlier work, supported by this grant, we found that in certain parameter regimes associated with given initial data, spurious joint spatial-temporal numerical chaos was generated by truncation effects due to nonintegrable discretizations. The phenomena can be traced to the proximity of the initial data to underlying homoclinic manifolds inherent in the governing problem. We also developed a numerical algorithm which was superior, and which is based upon soliton theory. The latter scheme is referred to as the integrable discretization. For the nonintegrable discretization, the chaos was observed to disappear as the mesh was refined. In our recent work, just published in Physical Review Letters, we show that there are parameter regimes in which roundoff effects induce spurious numerical chaos. These roundoff induced instabilities are associated even with the best numerical schemes such as the Fourier split-step algorithm (used by many researchers) and the integrable discretization mentioned above. We have found the analytical reason for this phenomenon. It turns out to be due to very small effects in the eigenvalues of the associated linear scattering problem which governs the solution of the original problem via the Inverse Scattering Transform. A subclass of eigenvalues are indicative of homoclinic structures in the original PDE, and under perturbations, even on the order of roundoff, tiny changes in these eigenvalues translate into large changes in the solution of the PDE. We have carried out a relevant perturbation analysis of the scattering problem and have shown that this agrees with numerical computations. Further investigations will be published in the near future. We believe that the numerical instabilities and chaos mentioned above are, in fact typical of what can occur when the governing problem has underlying homoclinic manifolds and will be observed in a wide class of Hamiltonian wave systems, and not necessarily ones which are integrable.

## BOOKS:

1. *Solitons, Nonlinear Evolution Equations and Inverse Scattering*, M.J. Ablowitz and P.A. Clarkson, London Mathematical Society Lecture Notes Series #149, 516 pages, Cambridge University Press, New York, 1992 (US Edition).

## JOURNAL ARTICLES:

1. On the Method of Solution to the  $2 + 1$  Toda Equation, J. Villarroel and M.J. Ablowitz, *Phys. Lett. A*, **163** (1992) 293-296.
2. A New Hamiltonian Amplitude Equation Governing Modulated Wave Instabilities, M. Wadati, H. Segur and M.J. Ablowitz, *Journal of the Physical Society of Japan*, **61** (1992) 1187-1193.
3. Numerical Homoclinic Instabilities in the Sine-Gordon Equation, B.M. Herbst and M.J. Ablowitz, *Quaestiones Math.*, **15** (1992) 345-363.
4. Stable, Multi-State, Time-Reversible Cellular Automata with Rich Particle Content, M.J. Ablowitz, J.M. Keiser and L.A. Takhtajan, *Quaestiones Math.*, **15** (1992) 325-343.
5. Interaction Effects on Wavelength Multiplexed Soliton Data Packets, A.F. Benner, J.R. Sauer and M.J. Ablowitz, PAM<sup>†</sup> #129 (May 1992), to be published *J. Opt. Soc. of America*.
6. A Self-Dual Yang-Mills Hierarchy and its Reductions to Integrable Systems in  $1 + 1$  and  $2 + 1$  Dimensions, M.J. Ablowitz, S. Chakravarty and L.A. Takhtajan, PAM<sup>†</sup> #132 (August 1992), to be published *Commun. Math. Phys.*
7. Numerical Chaos, Symplectic Integrators and Exponentially Small Splitting Distances, by B.M. Herbst and M.J. Ablowitz, *J. Comp. Phys.*, **105** (1993) 122-132.
8. On the Inverse Scattering Transform of the  $2 + 1$  Toda Equation, J. Villarroel and M.J. Ablowitz, *Physica D*, **65** (1993) 48-70.
9. Numerical Chaos, Roundoff Errors and Homoclinic Manifolds, M.J. Ablowitz, C. Schober, and B.M. Herbst, *Phys. Rev. Lett.*, **71** (1993) 2683-2686.
10. Solutions to the  $2 + 1$  Toda Equation, J. Villarroel and M.J. Ablowitz, PAM<sup>†</sup> #169 (August 1993), to be published *J. Phys. A*.
11. On the Method of Solution of the Differential-Delay Toda Equation, J. Villarroel and M.J. Ablowitz, *Phys. Lett. A*, **180** (1993) 413-418.

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## CHAPTERS IN BOOKS:

1. One Dimensional Reductions of Self-Dual Yang-Mills Fields and Classical Equations, S. Chakravarty, M.J. Ablowitz and P.A. Clarkson, in *Recent Advances in Relativity—Essays in Honor of Ted Newman*, pp 60-71, Eds. Allan I. Janis and John R. Porter, Birkhauser, Boston (1992).
2. On Reductions of Self-Dual Yang Mills Equations, S. Chakravarty and M.J. Ablowitz, in *Painlevé Transcendents*, pp 331-343, Eds. D. Levi and P. Winternitz, Plenum, New York (1992).
3. Self Dual Yang-Mills Equation and New Special Functions in Integrable Systems, S. Chakravarty, M.J. Ablowitz and L.A. Takhtajan, in *Nonlinear Equations and Dynamical*

*Systems*, pp 3-11, Eds. M. Boiti, L. Martina and F. Pempinelli, World Scientific, Singapore (1992).

4. Integrable Systems, Self-Dual Yang-Mills Equations and Connections with Modular Forms, M.J. Ablowitz, S. Chakravarty and L.A. Takhtajan, in *Nonlinear Problems in Engineering and Science*, pp 1-15, Eds. S. Xiao and X.-C. Hu, Science Press, Beijing, China (1992).
5. Solitons and Computation, M.J. Ablowitz and B.M. Herbst, in *Important Developments in Soliton Theory*, Eds. A. Fokas, V. Zakharov, Series in Nonlinear Dynamics, Springer-Verlag, Berlin (1993).
6. On the Elliptic 2 + 1 Toda Equation, J. Villarroel and M.J. Ablowitz, PAM<sup>†</sup> #140 (November 1992), to be published *Proc. of NATO Conference on Solitons and Nonlinear Evolution Equations*, Exeter, U.K.
7. Remarks on the Inverse Scattering Transform Associated with Toda Equations, M.J. Ablowitz and J. Villarroel, PAM<sup>†</sup> #176 (September 1993), to be published *Proc. of 1993 Conference on Inverse Scattering*, Bad Honnef, Germany.

#### PREPRINTS:

1. Soliton Eigenvalue Decomposition Using Frequency Domain Data, A.F. Benner, M.J. Ablowitz and J.R. Sauer, PAM<sup>†</sup> #128 (May 1992).
2. Rapidly Forced Burgers Equation, M.J. Ablowitz and S. DeLillo, PAM<sup>†</sup> #136 (October 1992).
3. Wave Collapse and Instability of Solitary Waves of a Generalized Nonlinear Kadomtsev-Petviashvili Equation, X.P. Wang, M.J. Ablowitz, and H. Segur, PAM<sup>†</sup> #146 (March 1993).
4. Parametric Forcing, Bound States and Solutions of a Nonlinear Schrödinger Type Equation, M.J. Ablowitz and S. De Lillo, PAM<sup>†</sup> #157 (May 1993).
5. Solutions to the 2+1 Toda equation, J. Villarroel and M.J. Ablowitz, PAM<sup>†</sup> #169 (August 1993).
6. On the method of solution of the differential-delay Toda equation, J. Villarroel and M.J. Ablowitz, PAM<sup>†</sup> #174 (August 1993).
7. Remarks on the Inverse Scattering Transform Associated with Toda Equations, M.J. Ablowitz and J. Villarroel, PAM<sup>†</sup> #176 (September 1993).
8. Homoclinic Manifolds and Numerical Chaos in the Nonlinear Schrödinger Equation, M.J. Ablowitz and C. Schober, PAM<sup>†</sup> #180 (October 1993).
9. Effective Chaos in the Nonlinear Schrödinger Equation, M.J. Ablowitz and C. Schober, PAM<sup>†</sup> #182 (November 1993).
10. Hamiltonian Integrators for the Nonlinear Schrödinger Equation, M.J. Ablowitz and C. Schober, PAM<sup>†</sup> #183 (December 1993).
11. Numerical Stochasticity, Hamiltonian Integrators and the Nonlinear Schrödinger Equation, M.J. Ablowitz and C. Schober, PAM<sup>†</sup> #184 (December 1993).

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#### INVITED PRESENTATIONS:

1. University of Kansas, Mathematics Department, "Chaos in the Nonlinear Schrodinger Equation: Real or Fake," Jan. 30, 1992; "On Reductions of the Self-Dual Yang-Mills Equations," Jan. 31, 1992.
2. Carlton University, Department of Mathematics, Ottawa, Canada, "Solitons, Computation and Chaos," April 11, 1992.
3. Kansas State University, Midwest Geometry Conference, "Reductions of the Self-Dual Yang-Mills Equations and New Integrable Systems," May 1-3, 1992.
4. SIAM Forum on Applied Mathematics, Moderator of Panel: "Training in Applied Mathematics/Preparation for Academic Careers," May 15-17, 1992.
5. Canadian Applied Math. Society, University of Alberta, Edmonton, Canada, Conference on Wave Phenomena, "Nonlinear Waves, Integrable Systems and Modular Forms," June 15-18, 1992.
6. NATO Advanced Research Workshop, Exeter University, Exeter U.K.; "Chaos in the Nonlinear Schrodinger Equation- Can it be Avoided?," July 14-18, 1992.
7. SUNY Stony Brook, Department of Mathematics, "Nonlinear Waves, Integrable Systems and a Novel Class of ODE's", December 3, 1992.
8. Challenge of Teaching Science and Mathematics, Graduate Teacher Program, University of Colorado at Boulder, "Coherence and Chaos", January 11, 1993.
9. Colorado School of Mines, Golden, Colorado, Sigma Xi lecture, "Solitons and Coherent Structures", March 30, 1993.
10. International Conference on Inverse Scattering, Bad Honnef, Germany, "Inverse Scattering and Nonlinear Wave Equations,  $1 + 1$ ,  $2 + 1$  and Higher Dimensions", May 17-20, 1993.
11. NATO Lectures, Bilkent University, Department of Mathematics and Faculty of Science, Ankara, Turkey, "Coherence and Chaos—Illustrative Case Studies"; "Reductions of Self Dual Yang Mills and New Special Functions in Integrable Systems", May 20-25, 1993.
12. Conference: Nonlinear Evolution Equation, Solitons and the Inverse Scattering Transform, Oberwolfach, Germany, "Numerical Chaos Roundoff Errors and Homoclinic Manifolds", July 11-17, 1993.
13. FSU-USA Conference on Chaos, NAS Center, Woods Hole, Mass., "Roundoff Errors and Homoclinic Manifolds", July 19-23, 1993.
14. Nonlinear Optics Workshop and Colloquium, Department of Mathematics, University of Arizona, "Roundoff Errors and Homoclinic Manifolds", Sept. 9-11, 1993.